Mat E 272-C
Fall 2001 – Homework Set #5
Callister – Chapter 8
SOLUTIONS TO ASSIGNED PROBLEMS and GRADING CRITERIA

GRADED problems: 8.3, 8.5, 8.10

SUGGESTED problems: 8.11, 8.14, 8.16, 8.24, 8.39, 8.40 (solutions provided at end of document)

8.3 What is the magnitude of the maximum stress that exists at the tip of an internal crack having a radius of curvature of 2.5×10^{-4} mm (10^{-5} in.) and a crack length of 2.5×10^{-2} mm (10^{-3} in.) when a tensile stress of 170 MPa (25,000 psi) is applied?

Equation (8.1b) can be used to solve this problem, as

$$\sigma_m = 2\sigma_o \left(\frac{a}{\rho_t}\right)^{1/2}$$

$$= (2)(170 \text{ MPa})\left[\frac{(2.5 \times 10^{-2} \text{ mm})/2}{2.5 \times 10^{-4} \text{ mm}}\right]^{1/2} = 2404 \text{ MPa} \ \ (354{,}000 \text{ psi})$$

Grading criteria: **3 points for correct equation (8.1b)**
1 point for correct substitution for variables
1 point for correct final answer.
5 points total

8.5 A specimen of a ceramic material having a modulus of elasticity of 300 GPa (43.5×10^6 psi) is pulled in tension with a stress of 900 MPa (130,000 psi). Will the specimen fail if its "most severe flaw" is an internal crack that has a length of 0.30 mm (0.012 in.) and a tip radius of curvature of 5×10^{-4} mm (2×10^{-5} in.)? Why or why not?

In order to determine whether or not this ceramic material will fail, you must determine its theoretical fracture (or cohesive) strength; if the maximum strength at the tip of the most severe flaw is greater than this value then fracture will occur--if less than, then there will be no fracture. The theoretical fracture strength is just **E**/10 (this is given in the lecture notes) or 30 GPa (4.35 x 10^6 psi), inasmuch as **E** = 300 GPa (43.5 x 10^6 psi).

The magnitude of the stress at the most severe flaw may be determined using Equation (8.1b) as

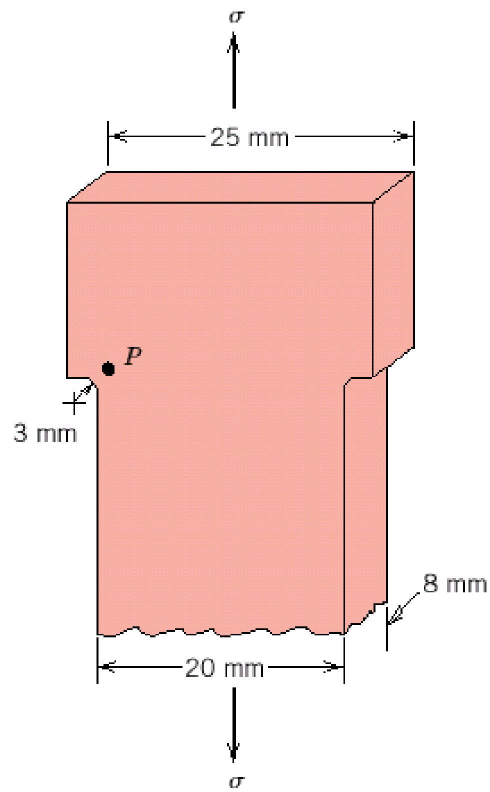
$$\sigma_m = 2\sigma_o \sqrt{\frac{a}{r_t}}$$

$$= (2)(900 \text{ MPa}) \sqrt{\frac{(0.3 \text{ mm})/2}{5 \times 10^{-4} \text{ mm}}} = 31.2 \text{ GPa} \quad (4.5 \times 10^6 \text{ psi})$$

Therefore, fracture **will** occur since this value is greater than **E**/10.

Grading criteria:
1 point for determination (or estimation) of fracture strength
3 points for use of correct equation (8.1b)
2 points for correct numerical result (31.2 GPa or 4.5E6 psi)
1 point for correct assessment of fracture (e.g., will occur)
7 points total

8.10. A portion of a tensile specimen is shown as follows:



(a) Compute the magnitude of the stress at point P when the externally applied stress is 100 MPa (14,500 psi).

(b) How much will the radius of curvature at point P have to be increased to reduce this stress by 20%?

(a) In this portion of the problem you need to determine the local stress at point **P** when the applied stress is 100 MPa (14,500 psi). In order to determine the stress concentration, it is necessary to use Figure 8.8c. From the geometry of the specimen, **w/h** = (25 mm)/(20 mm) = 1.25; furthermore, the **r/h** ratio is (3 mm)/(20 mm) = 0.15. Using the **w/h** = 1.25 curve in Figure 8.8c, the $\mathbf{K_t}$ value at **r/h** = 0.15 is 1.7. And since $\mathbf{K_t} = \frac{\sigma_\mathbf{m}}{\sigma_\mathbf{o}}$, then

$$\sigma_m = K_t\sigma_o = (1.7)(100\ \text{MPa}) = 170\ \text{MPa}\ (24{,}650\ \text{psi})$$

(b) Now it is necessary to determine how much **r** must be increased to reduce $\sigma_{\mathbf{m}}$ by 20%; this reduction corresponds to a stress of (0.80)(170 MPa) = 136 MPa (19,720 psi). The value of $\mathbf{K_t}$ is therefore, $\mathbf{K_t} = \dfrac{\sigma_{\mathbf{m}}}{\sigma_{\mathbf{o}}} = \dfrac{136\ \text{MPa}}{100\ \text{MPa}} = 1.36$. Using the **w**/**h** = 1.25 curve in Figure 8.8c, the value of **r**/**h** for $\mathbf{K_t}$ = 1.36 is about 0.43. Therefore

$$r = (0.43)h = (0.43)(20\ \text{mm}) = 8.60\ \text{mm}$$

Or, **r** must be increased from 3 mm to 8.6 mm in order to reduce the stress concentration by 20%.

Grading criteria:

2 points for determination of w/h (=1.25) and r/h (=0.15)
4 points for determination of local stress (=170 MPa)
1 point for calculation of desired local stress (=136 MPa)
1 point for determination of desired K_t (=1.36) from figure
2 points for correct radius (8.6 mm)
10 points total

Answers to Suggested Problems:

8.11 A cylindrical hole 25 mm (1.0 in.) in diameter passes entirely through the thickness of a steel plate 15 mm (0.6 in.) thick, 100 mm (4 in.) wide, and 400 mm (15.75 in.) long (see Figure 8.8*a*).

(a) Calculate the stress at the edge of this hole when a tensile stress of 50 MPa (7250 psi) is applied in a lengthwise direction.

(b) Calculate the stress at the hole edge when the same stress in part (a) is applied in a widthwise direction.

(a) This portion of the problem involves determination of the stress at the edge of a circular through-the-thickness hole in a steel sheet when a tensile stress is applied in a length-wise direction. We must use Figure 8.8a for **d**/**w** = $\frac{25 \text{ mm}}{100 \text{ mm}}$ = 0.25. From the figure and using this value, $\mathbf{K_t}$ = 2.4. Since $\mathbf{K_t} = \frac{\sigma_\mathbf{m}}{\sigma_\mathbf{o}}$ and $\sigma_\mathbf{o}$ = 50 MPa (7250 psi) then

$$\sigma_m = K_t\sigma_o = (2.4)(50 \text{ MPa}) = 120 \text{ MPa} \quad (17{,}400 \text{ psi})$$

(b) Now it is necessary to compute the stress at the hole edge when the external stress is applied in a width-wise direction; this simply means that **w** = 400 mm. The **d**/**w** then is 25 mm/400 mm = 0.0625. From Figure 8.8a, $\mathbf{K_t}$ is about 2.8. Therefore, for this situation

$$\sigma_m = K_t\sigma_o = (2.8)(50 \text{ MPa}) = 140 \text{ MPa} \quad (20{,}300 \text{ psi})$$

8.14 A specimen of a 4340 steel alloy having a plane strain fracture toughness of 45 MPa $\sqrt{\text{m}}$ (41 ksi $\sqrt{\text{in.}}$) is exposed to a stress of 1000 MPa (145,000 psi). Will this specimen experience fracture if it is known that the largest surface crack is 0.75 mm (0.03 in.) long? Why or why not? Assume that the parameter *Y* has a value of 1.0.

This problem asks us to determine whether or not the 4340 steel alloy specimen will fracture when exposed to a stress of 1000 MPa, given the values of $\mathbf{K_{Ic}}$, **Y**, and the largest value of **a** in the material. This requires that we solve for σ_c from Equation (8.13). Thus

$$\sigma_c = \frac{K_{Ic}}{Y\sqrt{\pi a}} = \frac{45 \text{ MPa}\sqrt{\text{m}}}{(1)\sqrt{(\pi)(0.75 \times 10^{-3} \text{ m})}} = 927 \text{ MPa} \ \ (133{,}500 \text{ psi})$$

Therefore, fracture will most likely occur because this specimen will tolerate a stress of 927 MPa (133,500 psi) before fracture, which is less than the applied stress of 1000 MPa (145,000 psi).

8.16 Suppose that a wing component on an aircraft is fabricated from an aluminum alloy that has a plane strain fracture toughness of 40 MPa $\sqrt{\text{m}}$ (36.4 ksi $\sqrt{\text{in.}}$). It has been determined that fracture results at a stress of 365 MPa (53,000 psi) when the maximum internal crack length is 2.5 mm (0.10 in.). For this same component and alloy, compute the stress level at which fracture will occur for a critical internal crack length of 4.0 mm (0.16 in.).

This problem asks us to determine the stress level at which an aircraft component will fracture for a given fracture toughness (40 MPa$\sqrt{\text{m}}$) and maximum internal crack length (4.0 mm), given that fracture occurs for the same component using the same alloy at one stress level and another internal crack length. It first becomes necessary to solve for the parameter **Y** for the conditions under which fracture occurred using Equation (8.11). Therefore,

$$Y = \frac{K_{Ic}}{\sigma\sqrt{\pi a}} = \frac{40 \text{ MPa}\sqrt{\text{m}}}{(365 \text{ MPa})\sqrt{(\pi)\left(\frac{2.5 \times 10^{-3} \text{ m}}{2}\right)}} = 1.75$$

Now we will solve for σ_c using Equation (8.13) as

$$\sigma_c = \frac{K_{Ic}}{Y\sqrt{\pi a}} = \frac{40 \text{ MPa}\sqrt{\text{m}}}{(1.75)\sqrt{(\pi)\left(\frac{4 \times 10^{-3} \text{ m}}{2}\right)}} = \mathbf{288 \ MPa} \ \ (41{,}500 \text{ psi})$$

8.24 Briefly explain why BCC and HCP metal alloys may experience a ductile-to-brittle transition with decreasing temperature, whereas FCC alloys do not experience such a transition.

With decreasing temperature, FCC metals do not experience a ductile-to-brittle transition because a relatively large number of slip systems remain operable even to very low temperatures. On the other hand, BCC and HCP metals normally exhibit this transition because the number of operable slip systems decreases with decreasing temperature.

8.39 List four measures that may be taken to increase the resistance to fatigue of a metal alloy.

Four measures that may be taken to increase the fatigue resistance of a metal alloy are:

1) Polish the surface to remove stress amplification sites.
2) Reduce the number of internal defects (pores, etc.) by means of altering processing and fabrication techniques.
3) Modify the design to eliminate notches and sharp contour changes.
4) Harden the outer surface of the structure by case hardening (carburizing, nitriding) or shot peening.

8.40 Give the approximate temperature at which creep deformation becomes an important consideration for each of the following metals: nickel, copper, iron, tungsten, lead, and aluminum.

Creep becomes important at $0.4\mathbf{T_m}$, $\mathbf{T_m}$ being the absolute melting temperature of the metal.

For Ni, $0.4\mathbf{T_m} = (0.4)(1455 + 273) = 691$ K or 418°C (785°F)
For Cu, $0.4\mathbf{T_m} = (0.4)(1085 + 273) = 543$ K or 270°C (518°F)
For Fe, $0.4\mathbf{T_m} = (0.4)(1538 + 273) = 725$ K or 450°C (845°F)
For W, $0.4\mathbf{T_m} = (0.4)(3410 + 273) = 1473$ K or 1200°C (2190°F)
For Pb, $0.4\mathbf{T_m} = (0.4)(327 + 273) = 240$ K or -33°C (-27°F)
For Al, $0.4\mathbf{T_m} = (0.4)(660 + 273) = 373$ K or 100°C (212°F)