

**CHAPTERS 9 & 10**  
**PHASE DIAGRAMS and TRANSFORMATIONS IN METALS**  
**SOLUTIONS to ASSIGNED PROBLEMS**  
**Maximum possible score: 36**

Assigned (and graded):

Chapter: 9.5 (a, b, c, & d only), 9.10 (part a only), 9.20, 9.30

Chapter 10: 10.2, 10.9, 10.10, 10.12

Suggested problems:

Chapter 9: 9.39, 9.45, 9.66, 9.57

Chapter 10: 10.14 (a, d, f, h), 10.16, 10.18, 10.19

---

**Assigned:**

**9.5** Cite the phases that are present and the phase compositions for the following alloys:

**(a)** 90 wt% Zn–10 wt% Cu at 400°C (750°F).

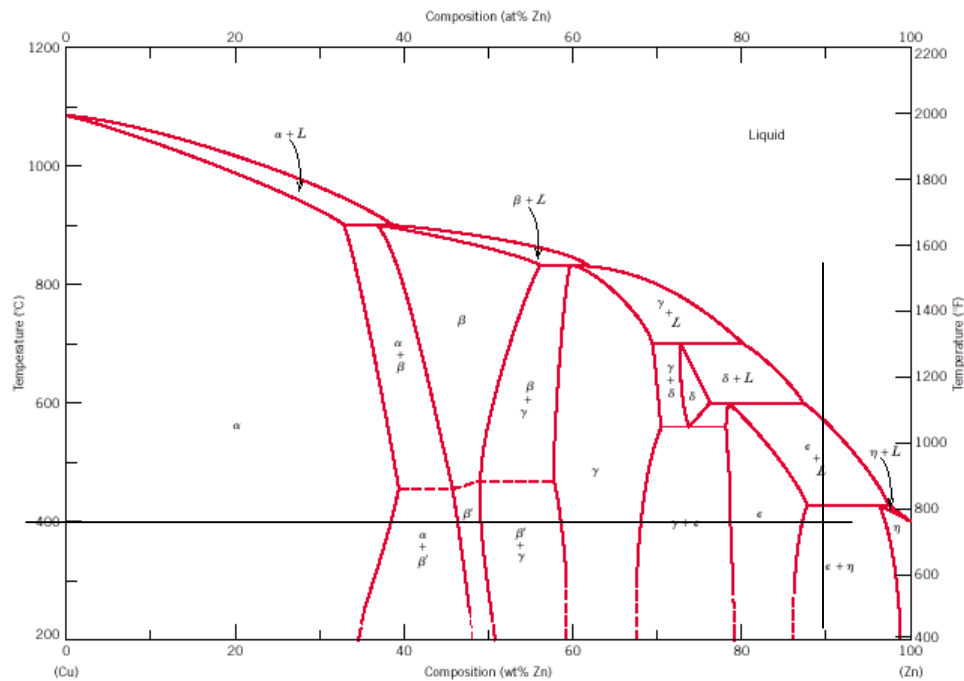
**(b)** 75 wt% Sn–25 wt% Pb at 175°C (345°F).

**(c)** 55 wt% Ag–45 wt% Cu at 900°C (1650°F).

**(d)** 30 wt% Pb–70 wt% Mg at 425°C (795°F).

This problem asks that we cite the phase or phases present for several alloys at specified temperatures.

(a)

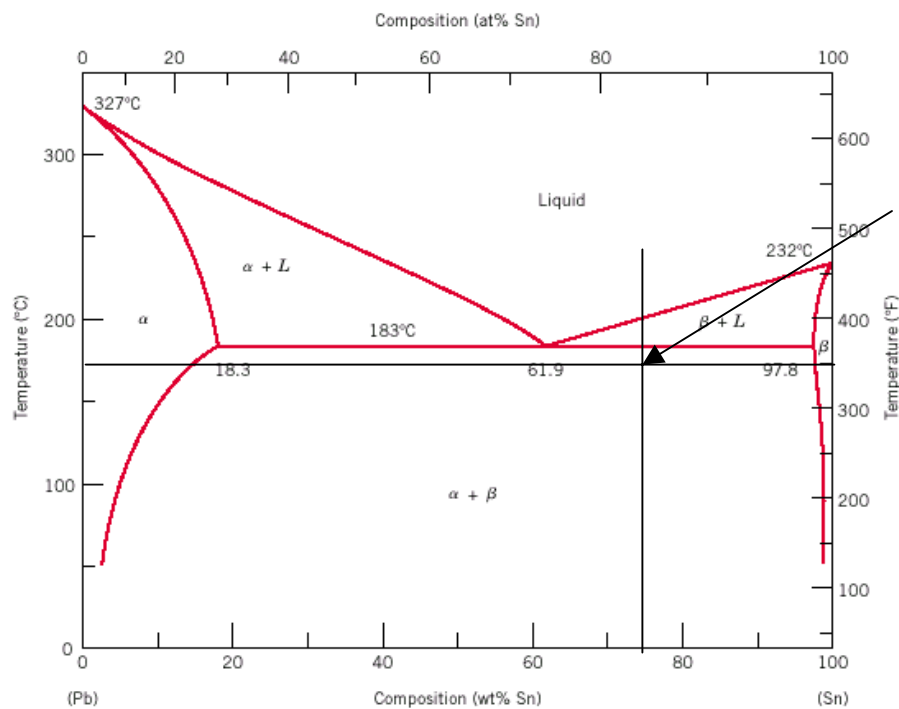


For an alloy composed of 90 wt% Zn-10 wt% Cu and at 400°C, from Figure 9.17,  $\epsilon$  and  $\eta$  phases are present, and

$$C_{\epsilon} = 87 \text{ wt\% Zn-13 wt\% Cu}$$

$$C_{\eta} = 97 \text{ wt\% Zn-3 wt\% Cu}$$

(b)

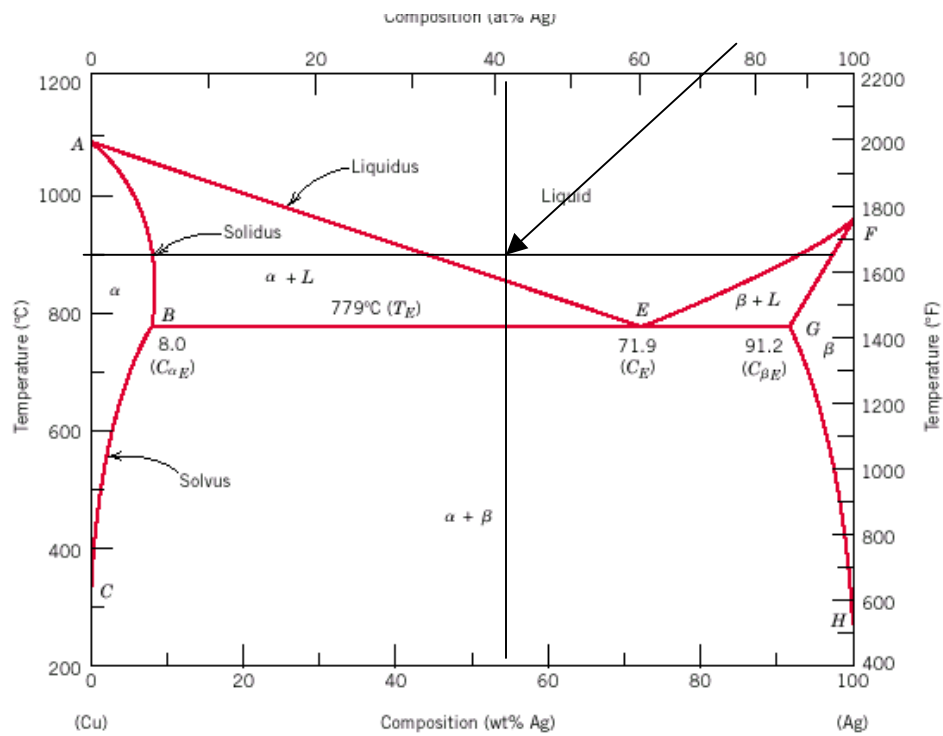


For an alloy composed of 75 wt% Sn-25 wt% Pb and at 175°C, from Figure 9.7,  $\alpha$  and  $\beta$  phases are present, and

$C_{\alpha} = 15 \text{ wt\% Sn-85 wt\% Pb}$

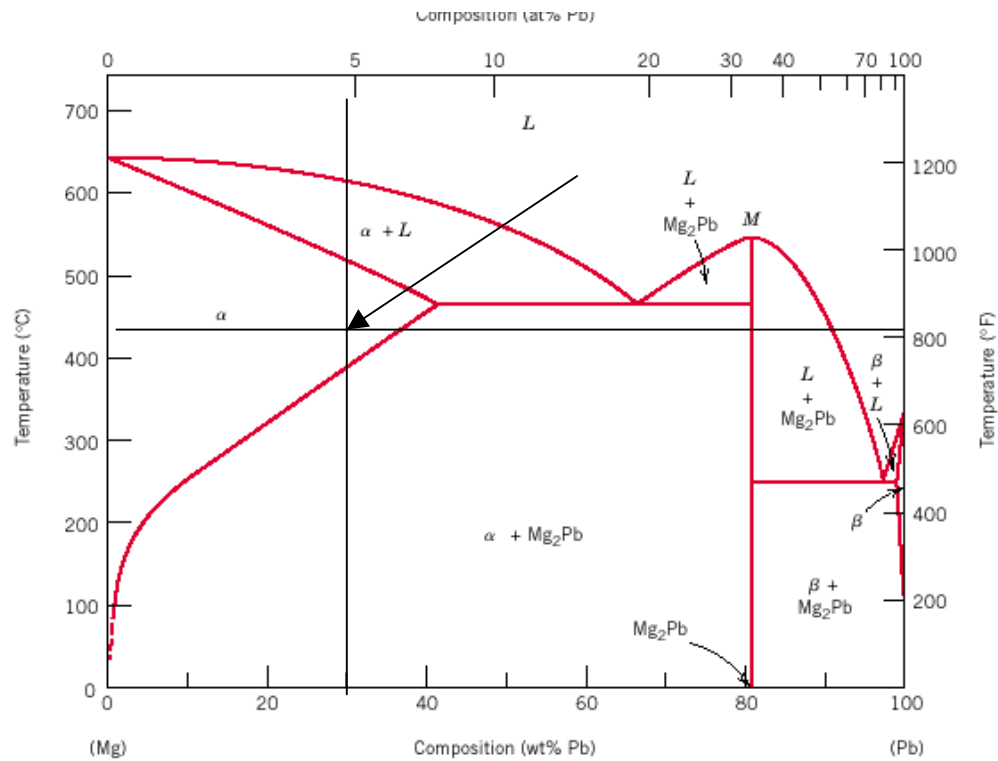
$C_{\beta} = 98 \text{ wt\% Sn-2 wt\% Pb}$

(c)



For an alloy composed of 55 wt% Ag-45 wt% Cu and at 900°C, from the figure, only the liquid phase is present; its composition is 55 wt% Ag-45 wt% Cu.

(d)

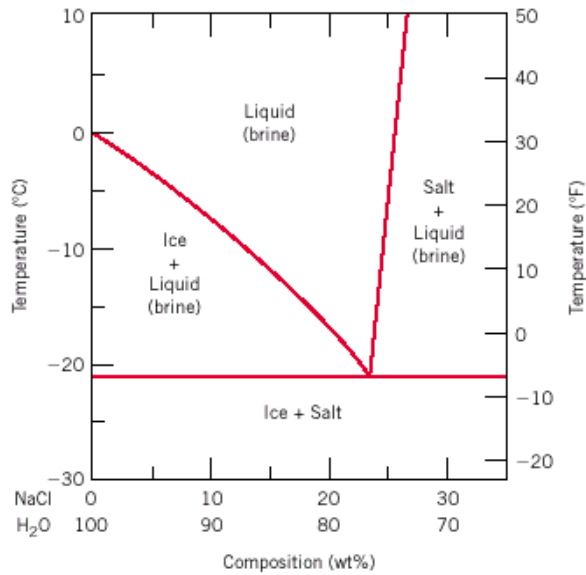


For an alloy composed of 30 wt% Pb-70 wt% Mg and at 425°C, from Figure 9.18, only the  $\alpha$  phase is present; its composition is 30 wt% Pb-70 wt% Mg.

Grading criteria: 1 point for correct identification of phase(s) present for each part  
1 point for correct identification of phase composition(s) for each part

**Total points possible for this problem: 8**

**9.10** Below is a portion of the  $\text{H}_2\text{O}$ – $\text{NaCl}$  phase diagram:



**(a)** Using this diagram, briefly explain how spreading salt on ice that is at a temperature below  $0^\circ\text{C}$  ( $32^\circ\text{F}$ ) can cause the ice to melt.

(a) Spreading salt on ice will lower the melting temperature, since the liquidus line decreases from  $0^\circ\text{C}$  to the eutectic temperature at about  $-21^\circ\text{C}$ . Thus, ice at a temperature below  $0^\circ\text{C}$  (and above  $-21^\circ\text{C}$ ) can be made to form a liquid phase by the addition of salt.

Grading criterion: 2 points for correctly recognizing that salt lowers the melting temperature.

**Total points possible for this problem: 2**

**9.20** A copper–nickel alloy of composition 70 wt% Ni–30 wt% Cu is slowly heated from a temperature of  $1300^\circ\text{C}$  ( $2370^\circ\text{F}$ ).

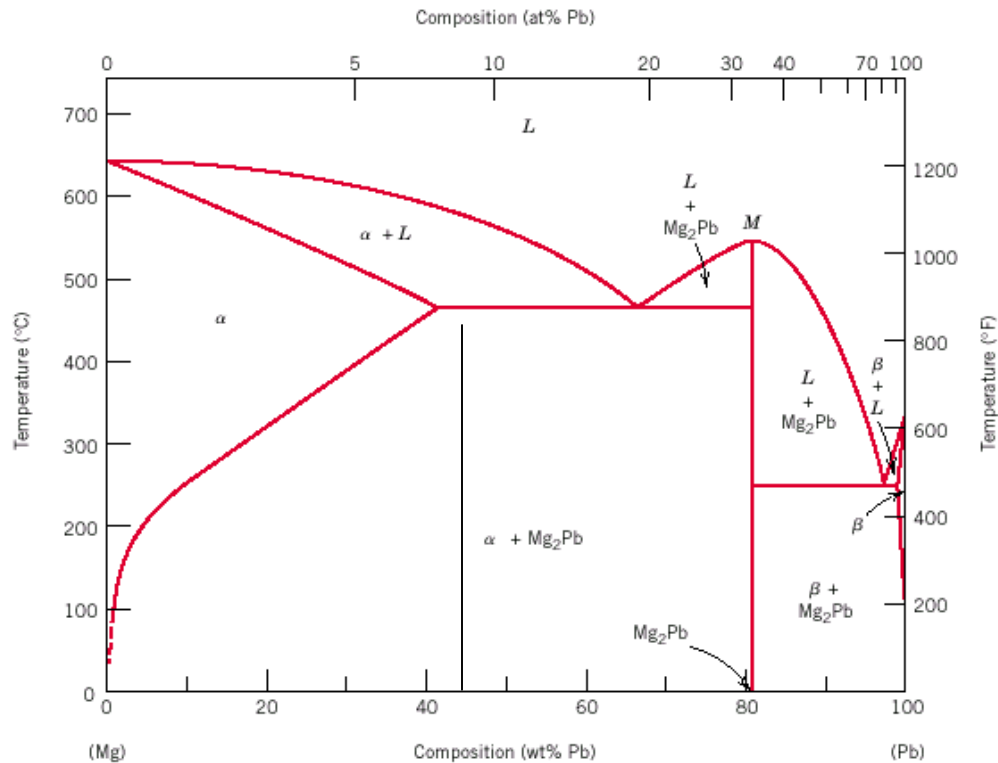
- (a)** At what temperature does the first liquid phase form?
- (b)** What is the composition of this liquid phase?
- (c)** At what temperature does complete melting of the alloy occur?
- (d)** What is the composition of the last solid remaining prior to complete melting?

Upon heating a copper–nickel alloy of composition 70 wt% Ni–30 wt% Cu starting from  $1300^\circ\text{C}$  and utilizing the appropriate phase diagram, Figure 9.2a:



**9.30** A 45 wt% Pb–55 wt% Mg alloy is rapidly quenched to room temperature from an elevated temperature in such a way that the high-temperature microstructure is preserved. This microstructure is found to consist of the  $\alpha$  phase and  $\text{Mg}_2\text{Pb}$ , having respective mass fractions of 0.65 and 0.35. Determine the approximate temperature from which the alloy was quenched.

We are asked to determine the approximate temperature from which a Pb-Mg alloy was quenched, given the mass fractions of  $\alpha$  and  $\text{Mg}_2\text{Pb}$  phases. We need to refer to the appropriate binary phase diagram, Fig. 9.18, p. 269, copied below:



We can write a lever-rule expression for the mass fraction of the  $\alpha$  phase as

$$W_{\alpha} = 0.65 = \frac{C_{\text{Mg}_2\text{Pb}} - C_o}{C_{\text{Mg}_2\text{Pb}} - C_{\alpha}}$$

The value of  $C_o$  is stated as 45 wt% Pb–55 wt% Mg, and  $C_{\text{Mg}_2\text{Pb}}$  is 81 wt% Pb–19 wt% Mg (you can read this directly off the phase diagram), which is independent of temperature; thus,

$$0.65 = \frac{81 - 45}{81 - C_{\alpha}}$$

which yields

$$C_{\alpha} = 25.6 \text{ wt\% Pb}$$

The temperature at which the  $\alpha$  - ( $\alpha$  +  $\text{Mg}_2\text{Pb}$ ) phase boundary (Figure 9.18) has a value of 25.6 wt% Pb is about 360°C (680°F). (acceptable range: 340 – 370 °C or 665 – 695 °F)

Grading criteria: use of lever law to obtain composition of alpha phase corresponding to the stated mass fraction of alpha: 5 points. “Reasonable” estimation of temperature corresponding to calculated alpha composition: 2 points.

**Total points possible for this problem: 7.**

(Note, many people had difficulties with this problem; it required use of the lever law to find the composition of a phase, rather than a mass fraction for which it was written. Just because an equation is written in such a way as to give the value of a particular physical quantity doesn't mean you can't use the same equation to find an unknown quantity on the *right* side of the equal sign.)

## Chapter 10:

**10.2** For some transformation having kinetics that obey the Avrami equation (Equation 10.1), the parameter  $n$  is known to have a value of 1.7. If, after 100 s, the reaction is 50% complete, how long (total time) will it take the transformation to go to 99% completion?

This problem calls for us to compute the length of time required for a reaction to go to 99% completion. It first becomes necessary to solve for the parameter  $k$  in the Avrami equation (Equation 10.1). Rearrangement of the Avrami equation and substitution of the stated values for 50% completion (i.e.,  $y = 0.50$ ) leads to

$$k = -\frac{\ln(1-y)}{t^n} = -\frac{\ln(1-0.5)}{(100 \text{ s})^{1.7}} = 2.76 \times 10^{-4}$$

Now, solving for the time to go to 99% completion

$$t = \left[ -\frac{\ln(1-y)}{k} \right]^{1/n}$$

$$\text{or, } t = \left[ -\frac{\ln(1 - 0.99)}{2.76 \times 10^{-4}} \right]^{1/1.7} = 305 \text{ s}$$

Grading criteria: 2 points if rewrite Avrami equation and substitute values corresponding to 50% completion in order to obtain the constant, k. Another 2 points if this value of k is substituted back in the equation to find time for 99% completion.

**Total points possible for this problem: 4**

- 10.9 (a)** Briefly describe the phenomena of superheating and supercooling.  
**(b)** Why do they occur?

(a) Superheating and supercooling correspond, respectively, to heating or cooling above or below a phase transition temperature without the occurrence of the transformation.

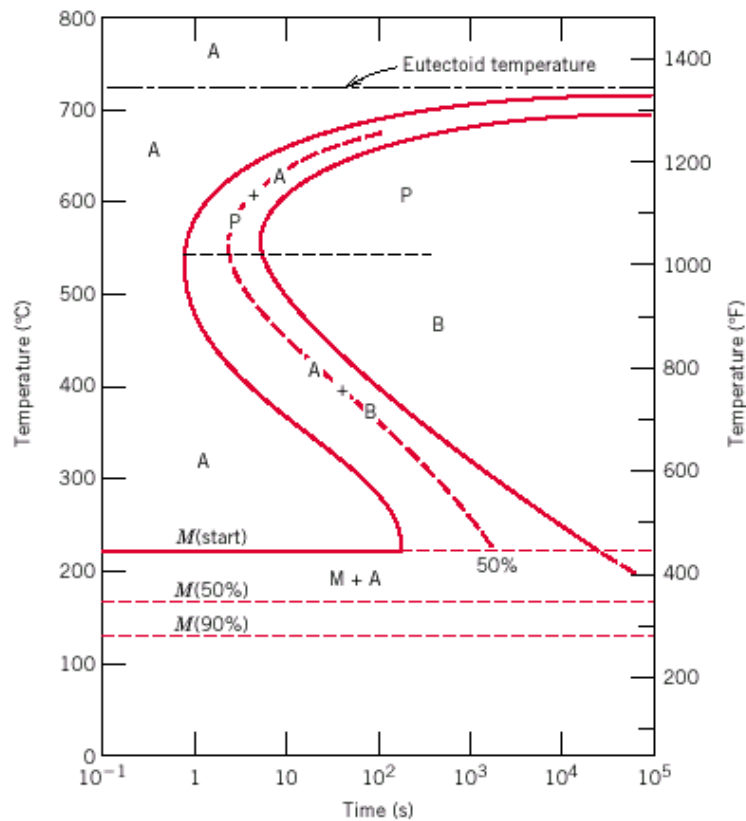
(b) They occur because right at the phase transition temperature, the driving force is not sufficient to cause the transformation to occur. The driving force is enhanced during superheating or supercooling.

Grading criterion: part a) 1 point for recognizing that the phenomena involve temperature excursions above or below the equilibrium phase transformation temperature. Part b) 1 point for recognizing that the tendency (or driving force) for transformation increases with superheating or supercooling.

**Total points possible for this problem: 2**

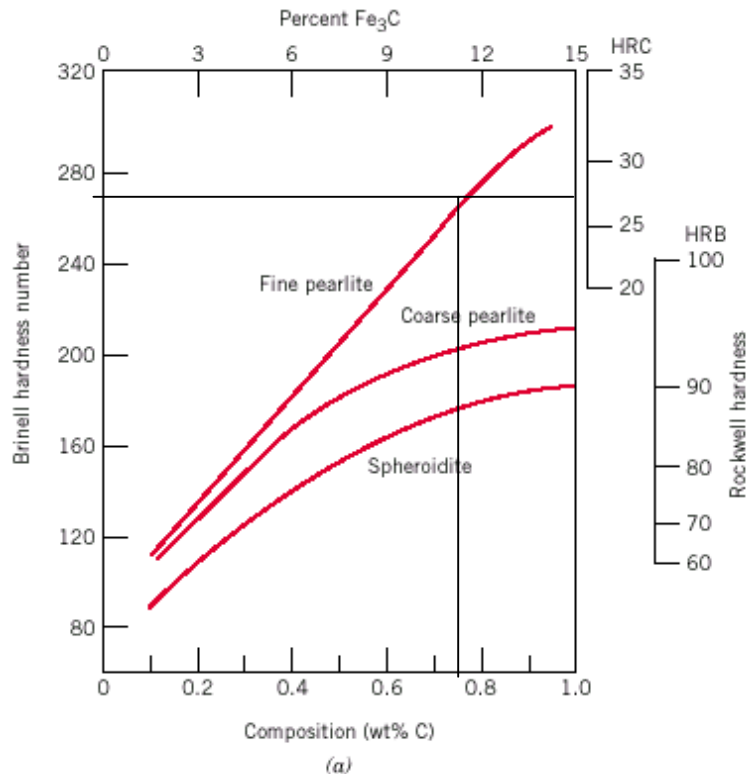
- 10.10** Suppose that a steel of eutectoid composition is cooled to 550°C (1020°F) from 760°C (1400°F) in less than 0.5 s and held at this temperature.  
**(a)** How long will it take for the austenite-to-pearlite reaction to go to 50% completion? To 100% completion?  
**(b)** Estimate the hardness of the alloy that has completely transformed to pearlite.

We are called upon to consider the isothermal transformation of an iron- carbon alloy of eutectoid composition (0.76 wt. % C ← that's what "eutectoid composition" means):



(a) From Figure 10.14, a horizontal line at 550°C intersects the 50% and reaction completion curves at about 2.5 and 6 seconds, respectively; these are the times asked for in the problem. (acceptable range: 1.5 to 3.5 seconds (50%), 4.5 to 7.5 seconds (completion))

(b) The pearlite formed will be **fine pearlite**. From Figure 10.22(a) (see next page), the hardness of an alloy of composition 0.76 wt% C that consists of fine pearlite is about **265 HB (27 HRC)**. (acceptable range: 240 to 280 HB)



Grading criteria: Part a) 1 point each for correct determination of transformation times (within stated acceptable range); Part b) 1 point for recognizing the need to use Figure 10.22a, 1 point for correct identification of fine pearlite microstructure, and 1 point for for correct estimation of hardness (within acceptable range).

**Total points possible for this problem: 5**

**10.12** Briefly cite the differences between pearlite, bainite, and spheroidite relative to microstructure and mechanical properties.

The microstructures of pearlite, bainite, and spheroidite **all consist of  $\alpha$ -ferrite and cementite phases**. For pearlite, the two phases exist as layers which alternate with one another. Upper bainite consists of very fine and parallel needles of ferrite that are separated by elongated particles of cementite; lower bainite consists of very thin plates of ferrite within which are situated very thin and parallel cementite particles. For spheroidite, the matrix is ferrite, and the cementite phase is in the shape of spheroidal-shaped particles.

Bainite is harder and stronger than pearlite, which, in turn, is harder and stronger than spheroidite.

Grading criteria: 1 point for recognizing that pearlite exists in layers (of ferrite and cementite); 1 point for stating that the structure of bainite (both upper and lower) is very fine (needles of ferrite in upper

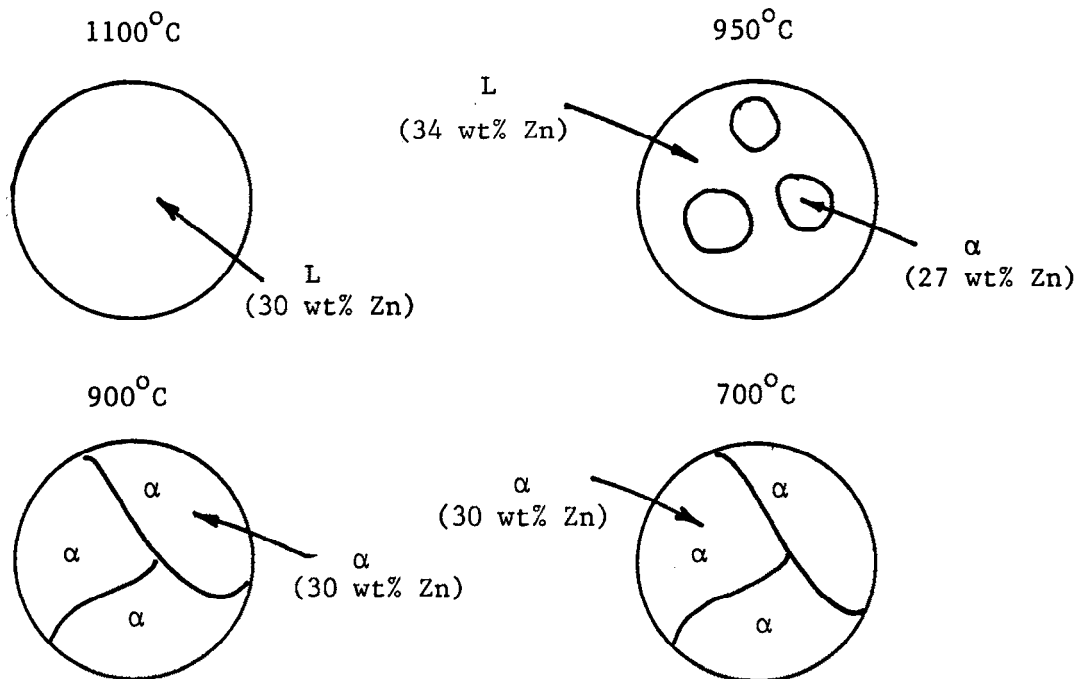
and thin plates for lower bainite); 1 point for stating that spheroidite consists of spherical-shaped particles of cementite. Finally, 1 point for correct order of hardness (or strength): spheroidite → pearlite → bainite.

**Total points possible for this problem: 4**

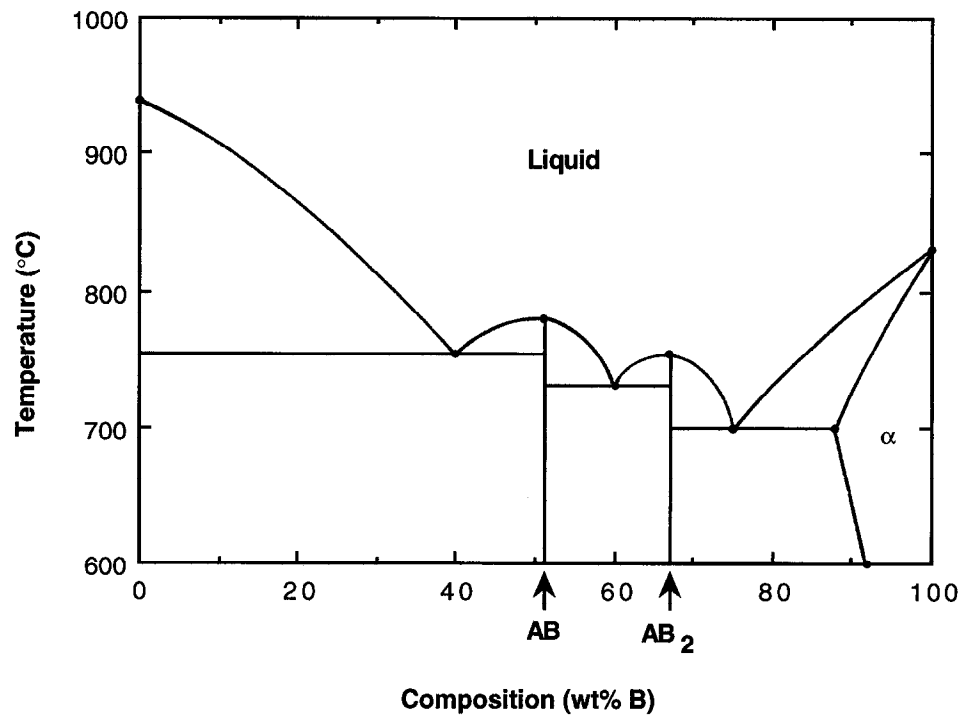
## Solutions to Suggested problems:

### Chapter 9:

9.39 Schematic sketches of the microstructures that would be observed for a 30 wt% Zn-70 wt% Cu alloy at temperatures of 1100°C, 950°C, 900°C, and 700°C are shown below. The phase compositions are also indicated.



9.45 Below is shown the phase diagram for these two A and B metals.

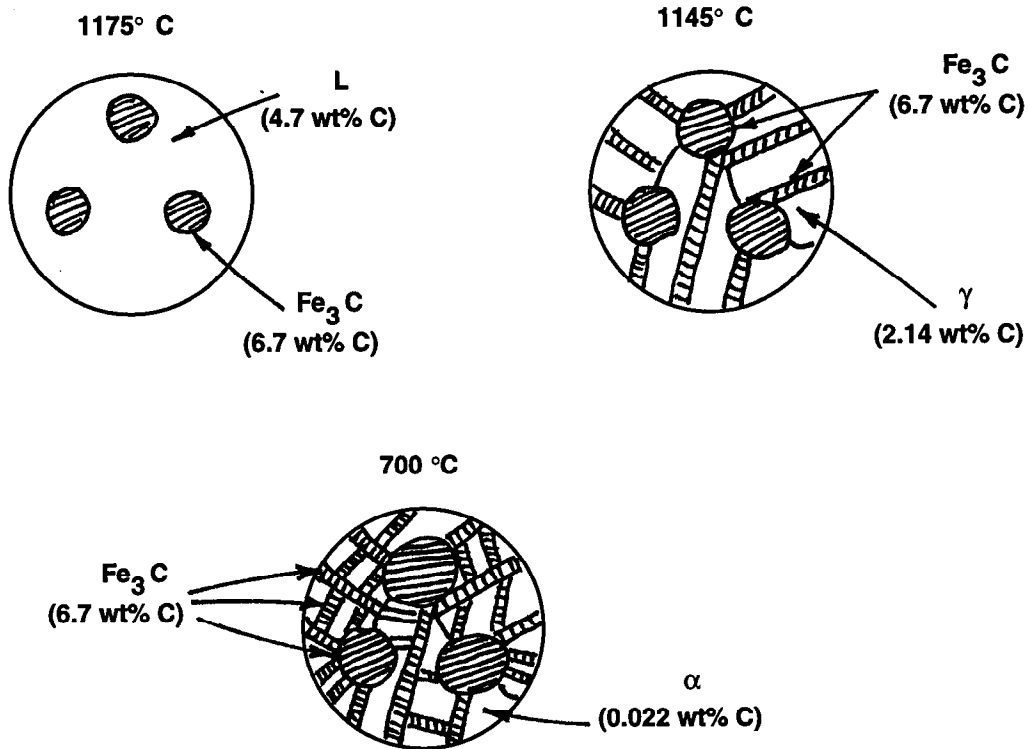


9.57 In this problem we are given values of  $W_{\alpha}$  and  $W_{Fe_3C}$  for an iron-carbon alloy (0.88 and 0.12, respectively) and then are asked to specify whether the alloy is hypoeutectoid or hypereutectoid. Employment of the lever rule for total  $\alpha$  leads to

$$W_{\alpha} = 0.88 = \frac{C_{Fe_3C} - C_0}{C_{Fe_3C} - C_{\alpha}} = \frac{6.70 - C_0}{6.70 - 0.022}$$

Now, solving for  $C_0$ , the alloy composition, leads to  $C_0 = 0.82$  wt% C. Therefore, the alloy is hypereutectoid since  $C_0$  is greater than 0.76 wt% C.

9.66 Schematic microstructures for the iron-carbon alloy of composition 5 wt% C-95 wt% Fe and at temperatures of 1175°C, 1145°C, and 700°C are shown below; approximate phase compositions are also indicated.



## Chapter 10:

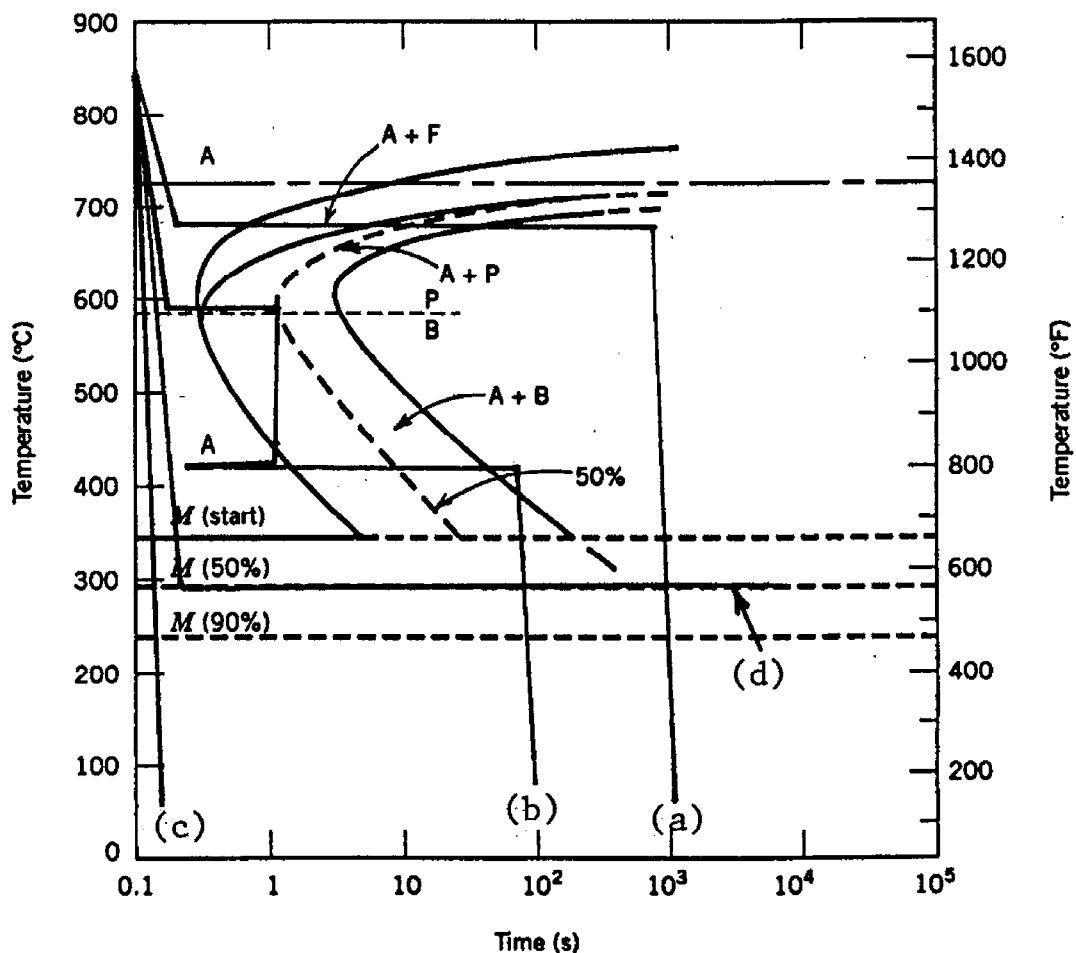
10.14 This problem asks us to determine the nature of the final microstructure of an iron-carbon alloy of eutectoid composition, that has been subjected to various isothermal heat treatments. Figure 10.14 is used in these determinations.

- (a) 50% coarse pearlite and 50% martensite
- (b) 100% spheroidite
- (c) 50% fine pearlite, 25% bainite (upper), and 25% martensite
- (d) 100% martensite
- (e) 40% bainite (upper) and 60% martensite
- (f) 100% bainite (upper)
- (g) 100% fine pearlite
- (h) 100% tempered martensite

10.16 We are asked to determine which microconstituents are present in a 0.45 wt% C iron-carbon alloy that has been subjected to various isothermal heat treatments.

- (a) Martensite
- (b) Proeutectoid ferrite and martensite
- (c) Bainite
- (d) Spheroidite
- (e) Ferrite, medium pearlite, bainite, and martensite
- (f) Bainite and martensite
- (g) Proeutectoid ferrite, pearlite, and martensite
- (h) Proeutectoid ferrite and fine pearlite

10.18 Below is shown an isothermal transformation diagram for a 0.45 wt% C iron-carbon alloy, with time-temperature paths that will produce (a) 42% proeutectoid ferrite and 58% coarse pearlite; (b) 50% fine pearlite and 50% bainite; (c) 100% martensite; and (d) 50% martensite and 50% austenite.



10.19 We are called upon to name the microstructural products that form for specimens of an iron-carbon alloy of eutectoid composition that are continuously cooled to room temperature at a variety of rates. Figure 10.19 is used in these determinations.

- (a) At a rate of  $200^{\circ}\text{C/s}$ , only martensite forms.
- (b) At a rate of  $100^{\circ}\text{C/s}$ , both martensite and pearlite form.
- (c) At a rate of  $20^{\circ}\text{C/s}$ , only fine pearlite forms.