CHAPTERS 5 & 6
DIFFUSION and MECHANICAL PROPERTIES-I
SOLUTIONS to ASSIGNED PROBLEMS

CHAPTER 5

5.7 We are asked to determine the **position** at which the nitrogen concentration is 2 kg/m^3. This problem is solved by using Equation (5.3) in the form

$$J = - D \frac{C_A - C_B}{x_A - x_B}$$

If we take $\mathbf{C_A}$ to be the point at which the concentration of nitrogen is 4 kg/m^3, then it becomes necessary to solve for $\mathbf{x_B}$, as

$$x_B = x_A + D\left[\frac{C_A - C_B}{J}\right]$$

Assume $\mathbf{x_A}$ is zero at the surface, in which case

$$x_B = 0 + (6 \times 10^{-11}\ m^2/s)\left[\frac{(4\ kg/m^3 - 2\ kg/m^3)}{1.2 \times 10^{-7}\ kg/m^2\text{-}s}\right]$$

$$= 1 \times 10^{-3}\ m = 1\ mm$$

5.11 We are asked to compute the diffusion **time** required for a specific nonsteady-state diffusion situation. It is first necessary to use Equation (5.5).

$$\frac{C_x - C_o}{C_s - C_o} = 1 - \operatorname{erf}\left(\frac{x}{2\sqrt{Dt}}\right)$$

wherein, $\mathbf{C_x}$ = 0.45, $\mathbf{C_o}$ = 0.20, $\mathbf{C_s}$ = 1.30, and $\mathbf{x}$ = 2 mm = 2 x 10^{-3} m. Thus,

$$\frac{C_x - C_o}{C_s - C_o} = \frac{0.45 - 0.20}{1.30 - 0.20} = 0.2273 = 1 - \mathrm{erf}\left(\frac{x}{2\sqrt{Dt}}\right)$$

or

$$\mathrm{erf}\left(\frac{x}{2\sqrt{Dt}}\right) = 1 - 0.2273 = 0.7727$$

It is acceptable to make the approximation erf (x) ≅ x. However, it is more accurate to determine the value of the argument by linear interpolation, using Table 5.1:

z	erf(z)
0.85	0.7707
z	0.7727
0.90	0.7970

$$\frac{z - 0.850}{0.900 - 0.850} = \frac{0.7727 - 0.7707}{0.7970 - 0.7707}$$

From which

$$z = 0.854 = \frac{x}{2\sqrt{Dt}}$$

Now, from Table 5.2, at 1000°C (1273 K)

$$D = (2.3 \times 10^{-5}\ \mathrm{m^2/s}) \exp\left[-\frac{148000\ \mathrm{J/mol}}{(8.31\mathrm{J/mol\text{-}K})(1273\ \mathrm{K})}\right]$$

$$= 1.93 \times 10^{-11}\ \mathrm{m^2/s}$$

Thus,

$$0.854 = \frac{2 \times 10^{-3}\ \mathrm{m}}{(2)\sqrt{(1.93 \times 10^{-11}\ \mathrm{m^2/s})\,(t)}}$$

Solving for **t** yields $t = 7.1 \times 10^{4}$ s = **19.7 h**

5.16 We are asked to compute the diffusion coefficients of C in both α and γ iron at 900°C. Using the data in Table 5.2,

$$D_\alpha = (6.2 \times 10^{-7} \text{ m}^2/\text{s}) \exp\left[- \frac{80000 \text{ J/mol}}{(8.31 \text{ J/mol-K})(1173 \text{ K})}\right]$$

$$= 1.69 \times 10^{-10} \text{ m}^2/\text{s}$$

$$D_\gamma = (2.3 \times 10^{-5} \text{ m}^2/\text{s}) \exp\left[- \frac{148000 \text{ J/mol}}{(8.31 \text{ J/mol-K})(1173 \text{ K})}\right]$$

$$= 5.86 \times 10^{-12} \text{ m}^2/\text{s}$$

The **D** for diffusion of C in BCC α iron is larger, the reason being that the atomic packing factor is smaller than for FCC γ iron (0.68 versus 0.74); this means that there is slightly more interstitial void space in the BCC Fe, and, therefore, the motion of the interstitial carbon atoms occurs more easily.

CHAPTER 6

6.5 This problem asks us to compute the **elastic modulus of steel**. For a square cross-section, $A_o = b_o^2$, where b_o is the edge length. Combining Equations (6.1), (6.2), and (6.5) and solving for E, leads to

$$E = \frac{Fl_o}{b_o^2 \Delta l} = \frac{(89000 \text{ N})(100 \times 10^{-3} \text{ m})}{(20 \times 10^{-3} \text{ m})^2 (0.10 \times 10^{-3} \text{ m})}$$

$$= 223 \times 10^9 \text{ N/m}^2 = \mathbf{223\ GPa} \quad (31.3 \times 10^6 \text{ psi})$$

6.7 (a) This portion of the problem calls for a determination of the **maximum load** that can be applied without plastic deformation (F_y). Taking the yield strength to be 275 MPa, and employment of Equation (6.1) leads to

$$F_y = \sigma_y A_o = (275 \times 10^6 \text{ N/m}^2)(325 \times 10^{-6} \text{ m}^2)$$

$$= \mathbf{89{,}375 \text{ N}} \ (20{,}000 \text{ lb}_f)$$

(b) The maximum length to which the sample may be deformed without plastic deformation is determined from Equations (6.2) and (6.5) as

$$l_i = l_o\left(1 + \frac{\sigma}{E}\right)$$

$$= (115 \text{ mm})\left[1 + \frac{275 \text{ MPa}}{115 \times 10^3 \text{ MPa}}\right] = \mathbf{115.28 \text{ mm}} \ (4.51 \text{ in.})$$

Or

$$\Delta l = l_i - l_o = 115.28 \text{ mm} - 115.00 \text{ mm} = \mathbf{0.28 \text{ mm}} \ (0.01 \text{ in.})$$

6.14 (a) We are asked, in this portion of the problem, to determine the **elongation** of a cylindrical specimen of aluminum. Using Equations (6.1), (6.2), and (6.5)

$$\frac{F}{\pi\left(\frac{d_o^2}{4}\right)} = E\,\frac{\Delta l}{l_o}$$

Or

$$\Delta l = \frac{4Fl_o}{\pi d_o^2 E}$$

$$= \frac{(4)(48800\ \mathrm{N})(200 \times 10^{-3}\ \mathrm{m})}{(\pi)(19 \times 10^{-3}\ \mathrm{m})^2(69 \times 10^9\ \mathrm{N/m^2})} = \mathbf{0.50\ mm}\ (0.02\ \mathrm{in.})$$

(b) We are now called upon to determine the change in diameter, Δd. Using the equation for Poisson's ratio (6.8)

$$\nu = -\frac{e_x}{e_z} = -\frac{\Delta d/d_o}{\Delta l/l_o}$$

From Table 6.1, for Al, $\nu = 0.33$. Now, solving for Δd yields

$$\Delta d = -\frac{n\Delta l d_o}{l_o} = -\frac{(0.33)(0.50\ \mathrm{mm})(19\ \mathrm{mm})}{200\ \mathrm{mm}}$$

$$= \mathbf{-1.6 \times 10^{-2}\ mm}\ \ (-6.2 \times 10^{-4}\ \mathrm{in.})$$

The diameter will decrease.